Currents in Stratified Water Bodies 3: Effects of Rotation

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Introduction

In this article, we outline the role of the Earth's rotation in modifying currents in inland waters. The first investigation into these dynamics was conducted by Lord Kelvin in the nineteenth century, and the analytical model he developed is still useful in describing some of these dynamics today. More recently, theoretical developments, laboratory experimentation, and field measurements have allowed for the development of a relatively complete picture of the role of the earth's rotation in inland waters.

It is important to note that the effect of the earth's rotation on currents in stratified lakes is predominantly through periodic, oscillatory motions such as gravity waves and vorticity waves, which will be defined later. It is not possible to discuss one without the other, and so in this article we discuss both waves and the currents they induce simultaneously, remembering that waves can be identified through fluctuations in potential energy (typically measured as thermocline or isotherm oscillations) or kinetic energy (typically measured as currents). It is therefore essential that the reader has a good understanding of the material presented in the preceding article. We follow on from this material by investigating how the wind induces motion in large lakes, where the rotation of the earth cannot be ignored.

We begin by defining several parameters that will assist in describing the impacts of the earth's rotation. The most important parameter is the Coriolis parameter (or the inertial frequency)

$$f = \frac{4\pi}{T}\sin\theta \qquad [1]$$

where T is the period of rotation of the earth (1 day or 86 400 s), θ is the latitude, and the units of f are radians per second. This parameter is zero at the equator (meaning that the effects of the earth's rotation on internal waves and currents can be ignored at the equator), and reaches a maximum value at the poles. The inertial period is defined as

$$T_I = \frac{2\pi}{f} \qquad [2]$$

which is infinite at the equator, and has a minimum value of 12 h at the poles. We also define the Rossby radius of deformation

$$R = \frac{c}{f} \qquad [3]$$

where c is the celerity (speed) of long gravity waves in the water body of interest, where we define long waves as those whose wavelength is far greater than the water depth. For surface ('barotropic') waves $c = \sqrt{gH}$, where g is the gravitational constant (9.8 m s^{-2}) and H is the water depth, as described in the preceding article.

Note that it is possible to represent a stratified system as an equivalent depth of homogenous fluid so that the internal ('baroclinic') dynamics can be represented by the same equations. For example, for a two-layer stratification, we can define the equivalent depth as

$$H_e = \frac{\rho_2 - \rho_1}{\rho_2}\frac{h_1 h_2}{h_1 + h_2} \qquad [4]$$

where ρ is the density and h the depth and the subscripts refer to the upper and lower layer. This allows for simple definition of the barotropic phase speed as $c = \sqrt{gH}$, and the baroclinic phase speed as $c_i = \sqrt{gH_e}$, and also allows us to define an internal Rossby radius of deformation that applies to baroclinic processes (those due to stratification) as

$$R_i = \frac{c_i}{f} \qquad [5]$$

The equivalent depth can also be defined for a continuous stratification, using c_m in eqn. [17] discussed later in this chapter.

We also define the Burger number

$$S = \frac{R}{L} \qquad [6]$$

where L is a length characterizing the basin length and/or width and R can represent either the Rossby radius or the internal Rossby radius. This dimensionless number is used to help classify both vorticity and gravity waves, and has been called by various names in the literature, such as the stratification parameter or nondimensional channel width. It is simply the ratio of the length scales at which rotation effects become important to the length scale of the lake in question, so for $S \to 0$ rotation is very important for the dynamics, and for $S \rightarrow \infty$ rotation can be ignored as the lake is physically small. Note that there is no abrupt transition, where the effects of rotation are suddenly felt at $S = 1$, but a gradual transition – this will be discussed later in this article.

Governing Equations

The dynamics described herein are based solely on the linear inviscid equations of motion for a homogenous fluid. The x-momentum, y-momentum, and conservation of mass equations are

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} \qquad [7]$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} \qquad [8]$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(Hu) + \frac{\partial}{\partial y}(Hv) = 0 \qquad [9]$$

where η is the height of the water surface above equilibrium, H is the water depth, and f is the Coriolis parameter. The momentum equations listed here are nothing more than the application of the Newton's famous equation $F = ma$, where the terms on the left-hand side represent acceleration terms (unsteady and Coriolis, respectively), and the term on the right-hand side represents the restoring force due to gravity. The same equations can be applied for barotropic and baroclinic motions, where for the baroclinic case, we replace the actual water depth H by the equivalent depth H_e described earlier.

In describing the effects of the earth's rotation on currents in inland waters, we consider two classes of motion based on the above equations. As we are interested in rotational effects, we first assume $f \neq 0$, such that we are sufficiently far away from the equator. For the first class of motions, which we will term 'gravity waves,' we also assume that the body of water under consideration is sufficiently small such that f can be considered constant (i.e., the lake is at a constant latitude) and that the bottom is flat, and therefore the restoring force is due to gravity only. For the second class of motions, which we will term 'vorticity waves,' we assume f is constant as for gravity waves, but we allow for variable water depth. This variable water depth allows for waves that arise due to the conservation of angular momentum. Dynamically, the second class of motions have similar characteristics to planetary Rossby waves in the ocean and atmosphere (i.e., where the latitude is not considered constant). In most cases, gravity waves dominate the dynamics of lakes and hence are explained later in detail. Only a brief summary description is given of the dynamics of vorticity waves, and for additional information the reader is referred to the references in Further Reading.

Gravity Waves

Of the two classes of periodic motions outlined earlier, gravity waves are the most well-studied and best understood in inland waters. We will consider only linear waves, that is, motions where the amplitude of the oscillations of the thermocline is small compared with the depth of the surface and bottom layer. This is not a major restriction on the analysis, as the inclusion of nonlinear effects has been shown in most cases to require only a minor correction to the linear approximation. In this article, we focus on cases where the Burger number is ~1, such that rotational effects can be expected. For surface (barotropic) waves, this would typically require lakes of more than 300 km width, of which there are very few. For baroclinic motions, where the phase speed c is far less than for barotropic motions, there are many lakes in which the Burger number is ~1. For typical values of the baroclinic internal wave phase speed (0.05–0.4 m s^{-1}), the internal Rossby radius is ~1–5 km (Figure 1), indicating that internal gravity waves in lakes of this scale (or larger) should experience the rotational effects of the earth. Note also that unlike the barotropic phase speed, which depends on water depth alone, the baroclinic phase speed varies as a function of stratification and so changes through the year. Rotation may therefore play a more important role in the internal wave dynamics during the strongly stratified period when the internal Rossby radius (and therefore the Burger number) are minimal than at other times of the year.

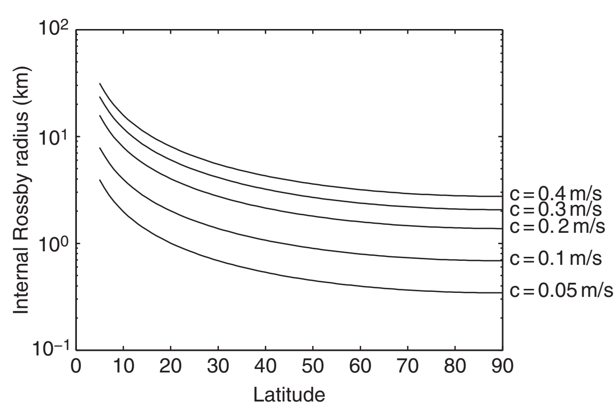

Figure 1 Internal Rossby radius as a function of latitude for several internal wave speeds. For horizontal length scales similar to or greater than the internal Rossby radius, rotational effects should be observed.

To understand the form of these motions in inland waters, it is instructive to build up our knowledge from simpler systems. We begin in a rotating system without boundaries, such as in the middle of the ocean far from the coast, where the classic gravity wave solutions are called plane progressive Poincaré waves. The amplitude (η) and velocity structure (u,v) associated with these waves can be described by

$$\eta = \eta_0 \cos(kx - \omega t) \quad [10]$$

$$u = (\omega \eta_0 / kH)\cos(kx - \omega t) \quad [11]$$

$$v = (f \eta_0 / kH)\sin(kx - \omega t) \quad [12]$$

where u is the velocity in the direction of propagation of the wave, v is the velocity in the transverse direction, η_0 is the maximum amplitude, k is the wave number (=2π/λ, where λ is the wavelength), ω is the wave frequency (=2π/T, where T is the wave period), H is the water depth (or equivalent water depth H_e for an internal wave), and f is the inertial frequency. The fluid particle trajectories (in plan view, as the vertical motion is small due to the linear wave assumption) are ellipses with major axes in the direction of propagation, with the ratio of the ellipse axes equal to ω/f and the direction of rotation anticyclonic (i.e., opposite to the direction of rotation of the earth). For short waves, which have high frequency, ω/f is large and so the trajectory ellipses are long and thin. Long waves are the opposite, with ω/f approaching one as the wave frequency is low and therefore the particle tracks are circular and trace out the well-known 'inertial circles' in the ocean (Figure 2). The radius of these circular tracks is U/f, which can be reformulated using eqns. [11] and [12] as η_0/kH.

An important aspect of the dynamics of internal waves influenced by the earth's rotation is that energy is generally not equally split between kinetic and potential forms. For the plane progressive Poincaré wave (Figure 2), the mean kinetic energy per unit area is

$$\mathrm{KE} = \frac{1}{4}\left(\frac{\omega^2 + f^2}{\omega^2 - f^2}\right)\rho g \eta_0^2 \quad [13]$$

where ρ is the water density, and the potential energy per unit area is

$$\mathrm{PE} = \frac{1}{4}\rho g \eta_0^2 \quad [14]$$

such that the ratio of potential to kinetic energy is

$$\frac{\mathrm{PE}}{\mathrm{KE}} = \frac{\omega^2 - f^2}{\omega^2 + f^2} \quad [15]$$

This indicates that waves with a frequency much greater than the inertial frequency will have a potential

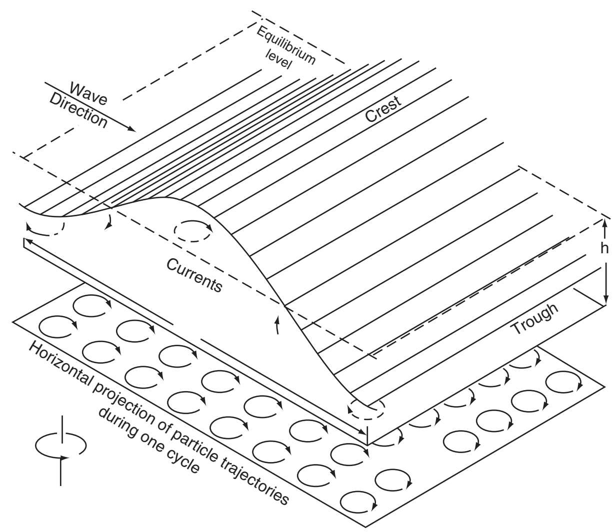

Figure 2 A long plane progressive Poincaré wave, in an infinite ocean, where $\omega \sim f$. Note the rotation of the current vectors is opposite to the direction of the earth's rotation. Reproduced from Mortimer CH (1974) Lake hydrodynamics. *Mitt. Int. Ver. Theor. Angew. Limnol.* 20: 124–197, with permission from E. Schweizerbart (http://www.schweizerbart.de/).

to kinetic ratio approaching one (as in the nonrotating case where $f \to 0$), and that waves close to the inertial frequency will have close to zero potential energy signal (such as the wave shown in Figure 2). This has implications for measurement of these waves, as they will only generally be observed by current measurements (a measure of kinetic energy) and not by fluctuations in stratification (a measure of potential energy variation).

The introduction of a boundary allows for the existence of Kelvin waves. The classical Kelvin wave solution is one in which the velocity perpendicular to the shore is considered to be zero (Figure 3). These waves propagate parallel to the boundary with the maximum amplitude at the shore, where the waves crests to the right (in the Northern Hemisphere) when looking along the direction of propagation. The amplitude decreases exponentially offshore at a rate equal to the Rossby radius of deformation R,

$$\eta = \eta_0 e^{-y/R} \cos(kx - \omega t) \quad [16]$$

where x is both the alongshore direction and the direction of propagation, and y is the offshore direction (Figure 3). Note that the phase speed of the wave is $c = \sqrt{gH}$, the same as for a wave in a nonrotating system. Current vectors, by definition, are rectilinear and oscillate in the alongshore direction only. As with waves in a nonrotating frame, the ratio of potential to kinetic energy is unity. For internal Kelvin waves, the dynamics are the same, except that the baroclinic phase speed applies and the wave amplitude decreases

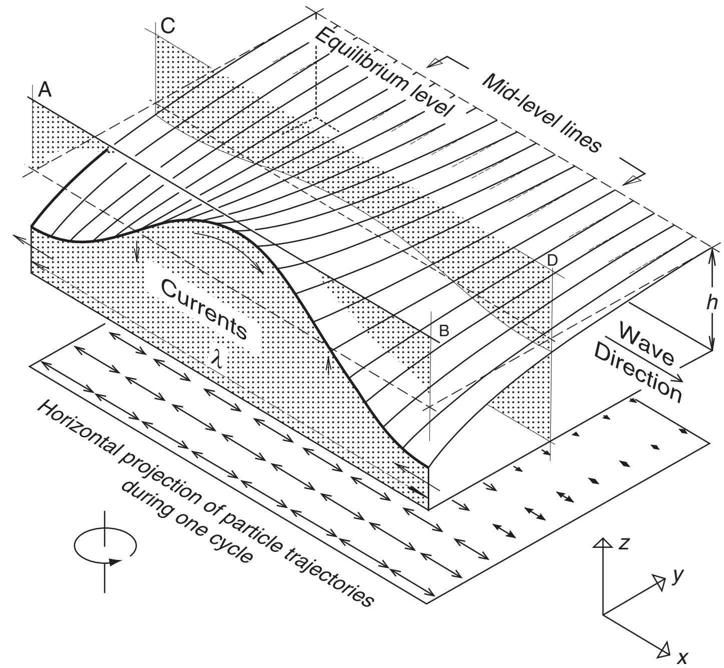

Figure 3 A long Kelvin wave progressing in the x-positive direction, with the shore located at $y=0$. Channel walls can be placed vertically at any point of constant *y*, for example indicated by the planes AB and CD. Reproduced from Mortimer CH (1974) Lake hydrodynamics. *Mitt. Int. Ver. Theor. Angew. Limnol.* 20: 124–197, with permission from • • • (http://www.schweizerbart.de/).

exponentially offshore with the internal Rossby radius of deformation R_i.

The introduction of a second boundary significantly complicates the waves supported in a rotating system. A channel (defined as two parallel walls with open ends) is able to support progressive Poincaré waves, made up of an obliquely incident plane progressive Poincaré wave with its reflection, and standing Poincaré waves, consisting of two progressive Poincaré waves traveling in opposite directions. These waves consist of cells similar to those presented in Figures 2 and 3, however the velocity at the border of each cell approaches zero. Closing a basin, and therefore creating a 'lake,' significantly complicates the wave field. For a rectangular basin, due to the complexity of the corners, an incident plus a reflected Kelvin wave is required along with an infinite number of Poincaré waves of the same frequency to satisfy the boundary conditions. Far simpler solutions can be found by assuming lakes to be represented by circular or elliptical basins of uniform depth, which we will use in the following discussion. As outlined earlier, a key nondimensional parameter controlling this response is the Burger number *S*. Based on this parameter, it is possible to determine the wave frequency for both circular and elliptic basins, the ratio of potential to kinetic energy in the wave response, and the response of a basin to external forcing. We will also rely on the simplified Kelvin and plane progressive Poincaré waves described earlier to assist in interpreting the results.

To understand the spatial structure of the waves (and the currents they induce) in a rotating system, it is helpful to consider the two end points: strong rotation ($S \to 0$) and no rotation ($S \to \infty$). For inland waters, these might also be considered the 'large lake' and 'small lake' case. For $S \to 0$, in the lake interior, we might expect plane progressive Poincaré waves to be present as outlined earlier (Figure 2), where the frequency approaches the inertial frequency, the current vectors rotate anticyclonically and the majority of energy is in the kinetic form. At the lake boundary, we might expect the classical Kelvin wave solution, where the offshore decay in amplitude is exponential at a rate *R*, the velocity at the boundary is parallel to the shore, the ratio of potential to kinetic energy is unity and the frequency approaches zero (Figure 3). Data collected from the North American Great Lakes support this conceptual model, with motion in the interior dominated by near-inertial frequencies and motion at the boundary appearing in the form of 'coastal jets,' which are the manifestation of the Kelvin wave solution. As the lake gets smaller (i.e., for $S \to \infty$), we should expect waves that are similar to the nonrotating case, where the ratio of potential to kinetic energy is unity, and the offshore decay of amplitude is no longer exponential, and the current vectors become rectilinear (i.e., the ellipses become long and thin).

How the characteristics of these waves vary as a function of Burger number and aspect ratio of the lake is graphically presented in Figure 4. The nondimensional frequency ω/f and the ratio of potential to kinetic energy are presented for analytical solutions to the circular and elliptical basin case for cyclonic (Kelvin-type) and anticyclonic (Poincaré-type) waves. Note that the potential to kinetic energy ratios are integrated over the entire lake, and do not represent the character at a particular point in space. We first consider waves in a circular basin (where the aspect ratio by definition is 1:1). In the strong rotation case ($S \to 0$), the cyclonic wave frequency goes to zero and the energy ratio approaches unity. This is the Kelvin wave limit for a semi-infinite boundary outlined earlier (Figure 3). For the anticyclonic wave, the frequency approaches the inertial frequency and the energy is predominantly kinetic – the plane progressive Poincaré wave solution outlined earlier (Figure 2). As S increases, the wave frequency for both types of waves increases and slowly converges as the lake gets smaller. The energy ratio for the anticyclonic wave also increases and asymptotically approaches unity, the nonrotation limit ($S \to \infty$). For the cyclonic wave, the energy ratio increases to a maximum of ~1.5, before asymptotically approaching unity as $S \to \infty$. Importantly as $S \to \infty$ these two solutions have the same characteristics (frequency, energy ratio, cross-basin structure), except that they rotate in opposite directions. They will thus manifest themselves at high S as a standing wave.

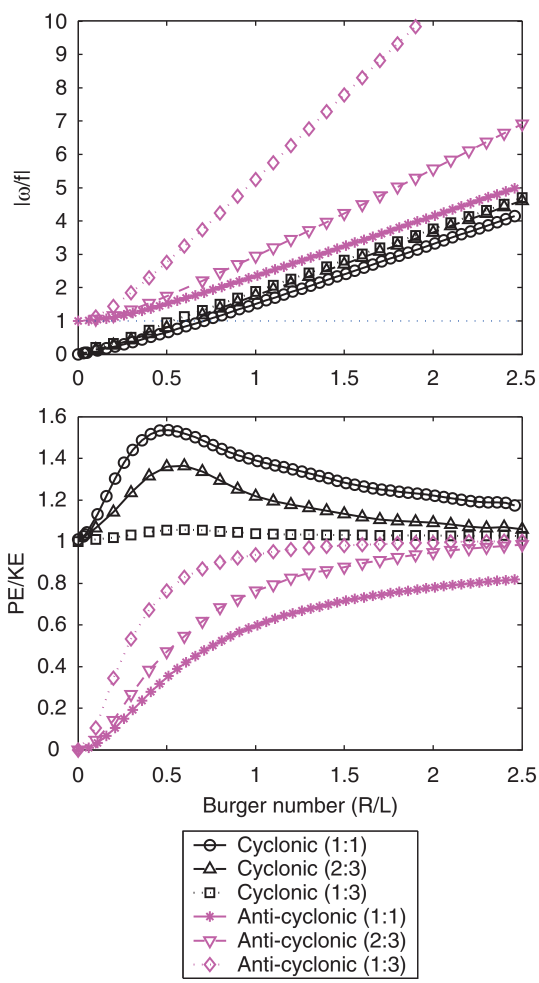

Figure 4 Nondimensional frequency (upper panel) and ratio of potential to kinetic energy (lower panel) as a function of wave type and aspect ratio, where the numbers in parentheses refer to the aspect ratio. The absolute value of the nondimensional frequency is presented as the results are independent of hemisphere. Reproduced from Antenucci JP and Imberger J (2001) Energetics of long internal gravity waves in large lakes. *Limnology and Oceanography* 46: 1760–1773, with permission from the American Society for Limnology and Oceanography.

The distribution of potential (i.e., thermocline oscillations) and kinetic energy (i.e., currents) in the basin also changes as the importance of rotation changes (Figure 5). For the strong rotation case ($S \to 0$), the cross-shore potential energy structure of the cyclonic waves has the exponential decay associated with Kelvin waves propagating along a shoreline, where we can rewrite eqn. [14] $\eta = \eta_0 e^{-y/SL}$ so that for small S the exponential decay is rapid relative to the lake width (Figure 5(a)). The kinetic energy is also predominantly located close to the shore (Figure 5(c)), hence the term 'coastal jet' being applied to these motions in the North American Great Lakes. As the importance of rotation decreases (S increases), there is a stronger signal of the cyclonic waves present in the interior. It is important to note that for this case, the currents are not parallel to the boundary everywhere in the lake – next to the shoreline they remain parallel as in Figure 3; however, towards the interior, the current ellipses become more circular and actually rotate in a cyclonic direction. For the anticyclonic (Poincaré-type) waves, the structure changes very little as the importance of rotation changes (Figure 5(b) and 5(d)). Note that as $S \to \infty$, the distribution of the potential and kinetic energy in the cyclonic wave approaches that of the anticyclonic wave.

We now consider the impact of changing the aspect ratio by moving towards elliptical basins from a circular basin shape. Note that the Rossby radius is defined in the elliptical basin based on the length of the major axis, not the minor axis, in Figure 4. The effect of decreasing the aspect ratio is that the system approaches the nonrotating case for lower values of S. Unlike the case of the circular basin, the frequencies of

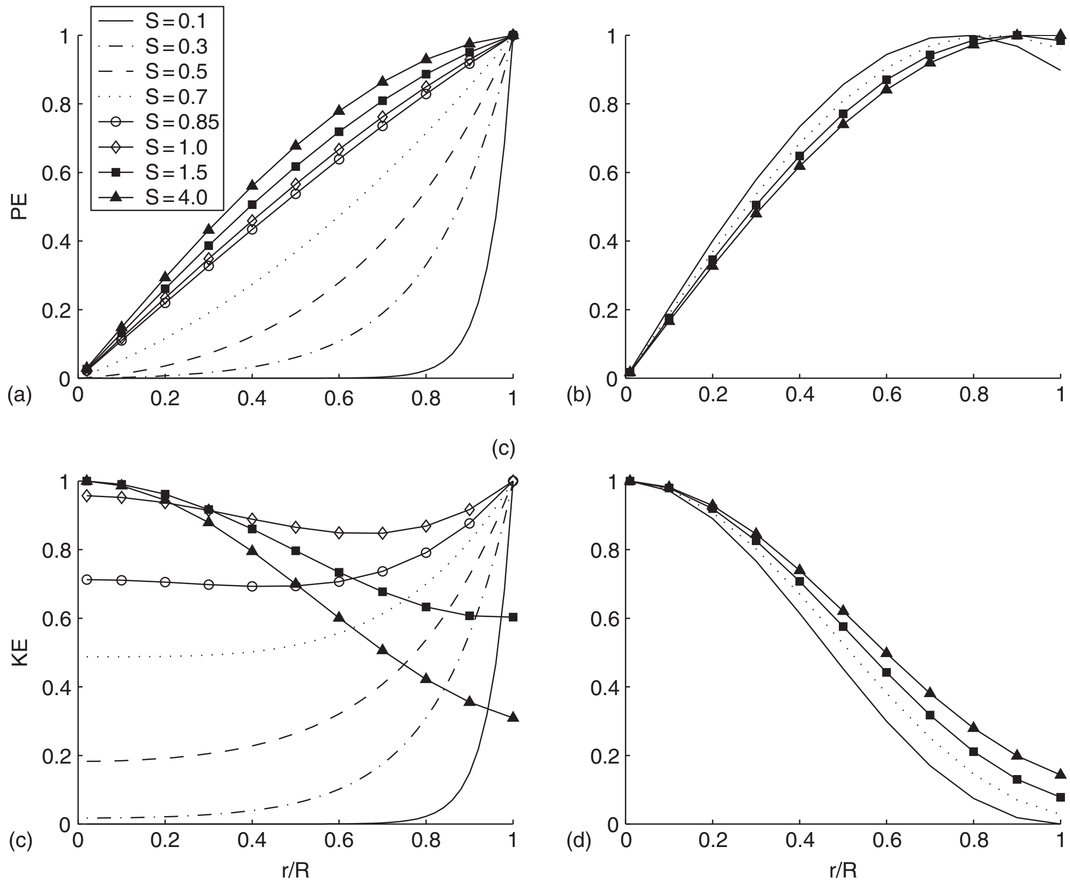

Figure 5 Radial structure of cyclonic and anticyclonic wave energy distribution as a function of Burger number for a circular lake for the lowest frequency motion (fundamental mode). The cyclonic wave structure is shown in panels (a) and (c). The anticyclonic wave structure is shown in panels (b) and (d). The radial structure for each Burger number has been normalized by its maximum value. Note that not all Burger numbers are shown in panels (b) and (d). Note the exponential decay in (a) is the same as that represented in **Figure 3** and eqn. [16]. Reproduced from Antenucci JP and Imberger J (2001) Energetics of long internal gravity waves in large lakes. *Limnology and Oceanography* 46: 1760–1773, with permission from the American Society for Limnology and Oceanography.

each cyclonic and anticyclonic wave pair diverge rather than converge. In the limit of $S \to \infty$, the cyclonic waves transform into longitudinal seiches, whereas the anticyclonic waves become the transverse seiche solution. It is for this reason that wind-forcing in the transverse direction has been observed to more easily generate anticyclonic, Poincaré-type, waves.

Current Structure and Measurement

As we have made the linear wave assumption, the vertical velocities induced by these motions are small; however, the horizontal current structure can show significant complexity both in the horizontal and vertical dimension. The complexity in the horizontal direction is due primarily to the presence of boundaries (and hence the horizontal structure of the waves), whereas the vertical complexity is due to both the stratification and the vertical mode.

It is important to tie the vertical position of the measurement location with the likely motion that dominates the flow. The simplest method to determine the likely points of maximum displacement and maximum current is to solve the long linear internal wave problem in a rotating system

$$\frac{d^2 w_m(z)}{dz^2} + \frac{N^2(z)}{c_m} w_m(z) = 0 \qquad [17]$$

where w_m is the vertical velocity eigenfunction for waves of vertical mode m, z the vertical dimension, $N^2(z)$ is the vertical profile of the square of the buoyancy frequency,

$$N(z)^2 = -\frac{g}{\rho_0}\frac{\partial \rho(z)}{\partial z} \qquad [18]$$

where g is the gravitational acceleration, ρ_0 is a reference density (typically the maximum density), $\rho(z)$ is the vertical profile of density, and c_m is the internal wave phase speed for the particular vertical mode in question. This is an eigenvalue problem, such that an infinite number of solutions exist for an infinite number of vertical modes m. For the case of a constant N^2, the equation has the sinusoidal solution

$$w_m(z) = \sin\left(\frac{m\pi}{H}z\right) \quad [19]$$

$$u_m(z) = u_m^0 \cos\left(\frac{m\pi}{H}z\right) \quad [20]$$

$$c_m = \frac{H}{m\pi}N \quad [21]$$

where u_m is the horizontal velocity induced by the wave and u_m^0 is a constant. It is quite clear from the above that the position of maximum vertical displacement is offset from the position of maximum horizontal current, which the selection of measurement points needs to take into account. Note that the equivalent depth for each vertical mode in a continuously stratified system can be calculated as $H_{em} = c_m/\sqrt{g}$.

For nonconstant $N^2(z)$, eqn. [17] is relatively easily solved numerically as it takes the form of a Sturm–Liouville equation, so from a depth profile of temperature a vertical profile of N^2 can be computed and fed into the eigenvalue solver, which will return both the eigenvalues c_m and the eigenvectors w_m. These eigenvectors will indicate which region of the water column will experience the maximum isotherm oscillations, and the derivate of this eigenvector with respect to z will give the position of the maximum horizontal velocity fluctuations. It is at these locations that thermistors and current meters should be concentrated, respectively. For most vertical modes, concentrating instruments in and around the thermocline is sufficient, except for capturing the velocity signal of the first vertical mode in which current measurements are best made either near the surface or bottom.

A key method to link currents with the wave motion described earlier is the use of rotary spectra of currents. By analyzing the different direction of propagation of the currents at different depths and points in space, it is possible to understand not only the predominant frequency of oscillation but also the predominant direction of rotation. Figure 6 shows data from Lake Kinneret in which the basin-scale wave field is dominated by a cyclonic vertical mode one Kelvin wave of period ~24 h, an anticyclonic vertical mode one Poincaré wave of period ~12 h, and an anticyclonic vertical mode two Poincaré wave of period ~22 h. Analysis of current data collected at this location indicates that the Kelvin wave effect on currents in the thermocline (where the 24 °C isotherm is located) is weak at this station as the current in the 20–24 h bandwidth is dominated by anticyclonic rotation.

Vorticity Waves

From the above equations, allowing either f to vary as a function of y (the β-plane) or allowing H to vary as a function of x and y allows a similar class of waves to exist. In the ocean, where f does vary, a class of waves called Rossby waves (or planetary Rossby waves) exist because of the conservation of angular momentum.

In inland waters such as lakes, these effects can be ignored as they are generally smaller than 500 km and f can be assumed to be constant. However, variations in water depth H result in a similar type of wave being possible, again due to the conservation of angular momentum. The structure of these motions is typically more complex than planetary Rossby waves as variations in water depth can occur in all directions, whereas variations in f are limited to the north–south

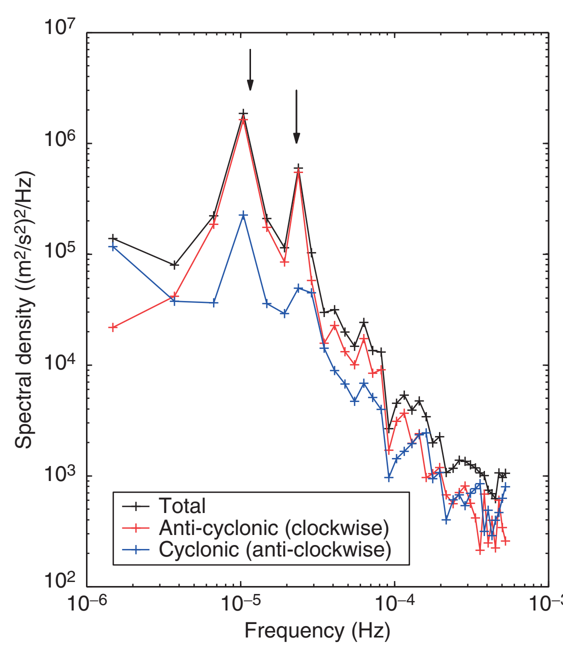

Figure 6 Spectra of currents along the 24C isotherm in Lake Kinneret during summer 1998 at station T3 on the western margin, showing the total spectrum, the component due to anticyclonic motion ('Poincaré waves'), and the component due to cyclonic motion ('Kelvin waves'). The arrows denote periods of 24 and 12 h from left to right. Adapted from Antenucci JP, Imberger J, and Saggio A (2000) Seasonal evolution of the basin-scale internal wave field in a large stratified lake. *Limnology and Oceanography* 45: 1621–1638.

direction. In this section, we use the term 'vorticity waves' to describe these motions, though they are also called 'topographic waves,' 'vortical modes,' 'second class waves,' or 'quasi-geostrophic waves.' These waves have been observed in large lakes such as Lake Ontario, Lake Michigan, Lake Zurich, and Lake Lugano.

The frequency of these motions is always subinertial (i.e., less than the inertial frequency at that latitude), and the frequency depends primarily on the topography of the basin as it is the topography (through the variation in $H(x,y)$) that causes changes in angular momentum. Importantly, the frequency of these motions is not a function of stratification, and so does not vary on a seasonal basis. This simplifies the measurement of these waves as they are existing at the same frequency year-round.

These waves propagate their phase cyclonically (anticlockwise in the Northern Hemisphere); however, the currents measured rotate both cyclonically and anticyclonically dependent on the horizontal structure of the wave. The currents induced by these waves in the bottom layer consist of a barotropic component only, whereas in the surface layer both a barotropic and a baroclinic current component exists, provided the upper layer is relatively thin than the lower layer. The exact structure of these waves is difficult to determine as there are multiple solutions that have similar frequencies, though it is recognized that the fundamental modes are the most likely to be generated.

Practical Guide to Measurement of these Waves

On the basis of the aforementioned disucssion, a step-by-step guide is provided on the method that should be applied when investigating these waves-

Gravity Waves:

1. Compute inertial frequency for the latitude of the lake in question using eqn. [1].
2. Make a simple two-layer approximation to the stratification, and compute the equivalent water depth H_e using eqn. [4].
3. Compute the internal wave speed using $c = \sqrt{gH_e}$. Typically this value will be between 0.1 and 0.3 $\mathrm{m\,s^{-1}}$.
4. Compute the internal Rossby radius using eqn. [5].
5. Based on the dimension of the lake, compute the Burger number using eqn. [6]. If the lake is approximately circular, use the radius for the length-scale L. If the lake is approximately elliptical, use the major axes half-length for the length-scale L. If the Burger number is greater than 2, rotational effects will be minimal. If the Burger number is less than 1, rotational effects will be very important.
6. From Figure 4(a), read off the nondimensional frequency ω/f for both the fundamental (lowest horizontal mode) cyclonic and anticyclonic wave for the aspect ratio of your lake.
7. Compute the angular frequency ω from the nondimensional frequency ω/f and the inertial frequency f for each of these two fundamental modes.
8. Compute the period T of these two waves from $T = 2\pi/\omega$.
9. Install measuring equipment (thermistor chain and/or current meters) for a sufficient period to measure more than 10 cycles of each wave. For example, if $T = 2$ days, at least 20 days of measurement will be required to achieve significant confidence in the data analysis. The location of these instruments should be carefully selected and is nontrivial. Typically the best location is halfway between the lake center and the lake boundary. Multiple sampling points are generally necessary – if two stations are deployed they should *not* be placed 180° apart as the direction of propagation can not be determined. It is best to orient stations such that they are 45–135° offset.
10. Compute spectra of temperature signals, isotherm depths, or integrated potential energy to determine the dominant frequencies in the field.
11. Compute rotary spectra of currents to determine the dominant rotation direction.
12. Compute phase and coherence between stations to assist in determining rotation direction and spatial structure. This can be done graphically by simply overlaying signals from the two stations or by using spectral analysis techniques.

Vorticity Waves:

1. Compute inertial frequency for the latitude of the lake in question using eqn. [1].
2. Deploy current meters for many periods longer than the inertial period, typically several months of record will be required. Two stations are required at the minimum, as with gravity waves they should *not* be placed 180° apart. Current meters should be placed in both the upper and lower layer, away from the thermocline.
3. Compute spectra of current signals to determine dominant frequencies, whether these frequencies change as a function of stratification, and whether the upper and lower layer current structure differs.

Armed with this information, it should be possible to determine which waves are dominating the temperature and current signals. Going into greater detail would typically require the reader to conduct further reading.

Glossary

Anticyclonic – Rotating in the opposite direction as the earth's rotation (clockwise in the Northern Hemisphere, anticlockwise in the Southern Hemisphere).

Baroclinic – A flow in which lines of constant pressure are not parallel with lines of constant density, in the context of lake dynamics this applies to all motion that is dependent on stratification.

Barotropic – A flow in which lines of constant pressure are parallel with lines of constant density, in the context of lake dynamics this assumption can be made when considering surface motion only.

Cyclonic – Rotating in the same direction as the Earth's rotation (anticlockwise in the Northern Hemisphere, clockwise in the Southern Hemisphere).

Equivalent depth – Depth of homogeneous fluid where the long-wave barotropic phase speed $c = \sqrt{gH_e}$ is equal to the long-wave baroclinic internal wave phase speed. As internal waves travel slower than surface waves, the equivalent depth is always smaller than the actual depth.

Gravity wave – Oscillatory motion where the restoring force is due to gravity.

Rectilinear motion – Movement in a straight line.

Vertical mode – Number of maxima in the vertical velocity structure for a particular wave.

Vorticity – The curl of the velocity field, or circulation ('spin') per unit area about a local vertical axis.

Vorticity wave – Oscillatory motion that results from the conservation of angular momentum in flow over varying topography.

Further Reading

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